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THE JOY OF SYMMETRY†

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It was a pleasant surprise for me one evening near the end of January in Budapest to receive a phone call from Jim Boggs. He told me that Herman Mark had withdrawn from being the banquet speaker, and instead, Jim was asking me to come over and give this speech. I was, of course, overwhelmed with joy. The question of topic came up, and Jim, knowing about my infatuation with symmetry, suggested symmetry as a possible general theme. By thought association I added my joy to it, hence the title of this presentation.

The importance of thought association will return later in this discourse. For the moment, however, let us remain with the question of this title. The joy part of it, for which I feel responsibility, is true. I was feeling happiness thinking about this evening while preparing for it in Budapest and I am feeling it now as well. I am, however, less than certain about the symmetry part of it. True, I like symmetry, but only to a certain extent. It may be nice, it may be pleasant, but having too much of it may also be boring and irritating. There are various statements pertaining to this, from relating too much symmetry to death, to simply saying that perfect symmetry is not suitable for human habitat. The opposite, i.e. asymmetry, may look more interesting, has more information content, but, of course this talk is not about asymmetry. Generally, however, I think that it is when symmetry is less than perfect, is somewhat distorted, that it really starts appealing to us.

I don't suppose that anybody is wondering how does the topic of symmetry come to a molecular structure meeting. It is one of the cementing concepts in science, and in particular in molecular sciences. My own interest in symmetry has certainly originated from my interest in the structure of molecules. I even remember the moment when I started wondering about symmetry and this wondering has not yet stopped. The biggest surprise I have experienced is the capability of this concept to relate such a diversity of fields and topics, and I am most grateful to it for opening up directions to me which I had not had interest in before, or to which I had even been hostile.

Some people may certainly be wondering, however, about the relationship between "The Joy of Symmetry" and this beautiful dinner party. The title of my presentation may remind us of the title of that bestseller *The Joy of Cooking*. When this comparison came to my mind I was a little discouraged, for cooking implies active participation whereas symmetry may imply passive observation rather than real involvement.

However, when we consider less than perfect symmetries, our participation is also needed. We are certainly not going to discuss geometrical symmetry tonight. Instead, we shall be dealing with symmetries which can only with a certain amount of magnanimity be called symmetries. This is a serious matter. The symmetries in chemical systems, for example, do not belong to geometrical symmetries but to what we may term material symmetries. It is our ability to geometrize, or to symmetrize, if you will, that makes the model of symmetry applicable to real systems. The rigor of geometrical symmetry is then gone and is replaced by some vagueness and fuzziness.

The Joy of Symmetry. This title may invite other thought associations as well. But then some people cannot be helped, as they thought-associate everything with what their mind is occupied by anyway. A few years ago I gave a talk about the importance of small structural differences in Edinburgh. Before the lecture, a distinguished chemist, with a name appearing in every monograph in his field, asked me, "This lecture of yours, will it be about sex?"

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One more thought about the history of this banquet speech; as you see, I find it difficult to get over this experience. A certain symmetry, or rather the asymmetry of flowing time, I feel in my role today. It was not very long ago, in 1971, that I was giving an invited talk to a crystallographic meeting in Manchester, England. Not only was it my first ever invited talk at a meeting, I also happened to be the youngest invited speaker at that meeting. Such a state of affairs, however, did not last long. In subsequent meetings, where I have happened to be an invited speaker, I no longer had the same distinction. Instead, I have always belonged to the medium age range of the speakers. Until this meeting, that is. Tonight I am once again in a unique state being the substitute speaker for 92-year-old Herman Mark.

For the benefit of those outside the electron diffraction field, Herman Mark was the initiator of gas-phase electron diffraction in 1930, and his associate Raymund Wierl carried out the first such experiment under his direction. Wierl, junior to Mark, died soon afterwards, but Mark survived and is very active even today. He has made a significant name in polymer sciences. He lives in Brooklyn, N.Y., and goes to work at Brooklyn Polytech every day when he is in town. Mark was born in 1895 in Vienna, lived in Germany at the time of starting gas electron diffraction, then returned to Vienna, and came to this country in the year of the Anschluss. His mother was born in Budapest, and Mark also visited Budapest as a small child at the end of the last century.

It happens that later in this speech I shall be making reference to another world known figure who himself was born in Hungary, then moved to Vienna, became a writer in the German language, then moved on westward and became a famous writer in the English language. The name is Arthur Koestler. With this I am not trying to say that all important people are Hungarian or of Hungarian origin, or at least related to Hungarians. Not at all. I have been told that other nations have also contributed to the progress of mankind. And, conversely, I would like to quote the statement which was displayed on the desk of Alexander Korda, the giant of the film industry. "It is not sufficient just to be Hungarian".

But let us return to our proper topic, symmetry. First of all, I would like to show you a beautiful Portugese stamp (Fig. 1) which displays a symmetrical pattern of a sixteenth century decoration. It was found on a tile. Such tilings are quite common on the facades of houses in Portugal. Besides, there is a whole museum of tiles in Lisbon. Let us, for a moment, disregard the central motif, and recognize the four mirror planes of the rest of the pattern. Including then the central motif into our consideration, the whole pattern is seen to have four-fold rotational symmetry only, and no mirror planes.

The medieval pattern on the Portugese stamp is surprisingly complete. Only some of the corners, and in particular the lower left edge show some damage. As we look at the pattern, and examine its symmetry, our eyes and mind skip involuntarily over the damages and imperfections, and see the pattern in its entirety, as a whole, and as if it were complete. This is exactly what I meant when I mentioned our ability to geometrize.

I find the patterns especially intriguing that have rotational symmetry only and no symmetry planes. I would like to show you yet another such pattern, again from Portugese tiles (Fig. 2). Whereas this pattern can be described by point group symmetry, its infinite repetition is described by space group symmetry (Fig. 3). As such patterns are often used for decorating walls, their space groups are even called wallpaper groups. There have been studies of how various space groups influence the mood of people, what is our perception of the various symmetries used for decorations. Some have argued that decorations possessing rotational symmetry only tend to

Fig. 1


Fig. 2

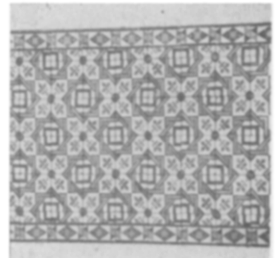
Fig. 3

Fig. 4

Fig. 5

Fig. 6

induce the feeling of motion in us, and facilitate moving even crowds of people. On the other hand, decorations with symmetry planes, especially symmetry planes perpendicular to each other, freeze us, prevent people from moving around. It has been suggeted that patterns with rotational symmetry only should be used for decorating dancing halls, and patterns with perpendicular symmetry planes should be used for decorating halls of important meetings.

A few years ago a fellow electron diffractionist, Shuzo Shibata gave me a beautiful Japanese slide on which there is a nice goat in front of the strikingly beautiful Mount Fuji (Fig. 4). I presume that anybody would find this picture of interest, but for a symmetrologist, this picture is a gold mine.

The goat has bilateral symmetry, and the same bilateral symmetry is characteristic for the whole animal kingdom as well as for humans. This bilateral symmetry originates from the fact that for animals, left and right are equivalent, whereas neither up and down, nor front and back are equivalent. This is the result of the translational motion along the Earth's surface so typical for animals. There are, of course, exceptions. I saw a double-headed dog a few years ago in Brussels (Fig. 5). It was not like the double-headed eagle of ancient coats of arms. Rather, this dog had a second head in place of its tail.

Mount Fuji, on the other hand, has cylindrical symmetry. All directions are equivalent around its vertical axis. There is no left and right, nor front and behind, only up and down are different. Many plants have the same cylindrical symmetry originating from their mode of life which fixes them to a certain place and lets them grow only upward.

Left and right symmetry is the most common of all symmetries, and in our everyday language it is often considered a synonym for symmetry. The conservation of left and right has for a long time been considered to be of fundamental importance in physics. We all have also learned about Buridan's ass who was prepared to exchange its life for the conservation of parity. However, more recently this parity has been shown to distort, and I have seen a German student poster advocating survival above parity (Fig. 6).

We find a lot of symmetry in architecture and municipal planning. The Tower of the University of Texas and the whole view of the central mall of the campus radiate symmetry (Fig. 7). What a relief then, especially on a hot summer day, when the operating fountain somewhat decreases this very high symmetry (Fig. 8). The beautiful new architecture in Dallas has certainly reduced symmetries without diminishing aesthetic pleasure (Figs 9 and 10).

At this point, I would like to introduce the most intriguing kind of symmetry, that is antisymmetry. For symmetry itself I did not even try to give a definition. Everybody knows what symmetry is. I myself used to think that I knew, until I started my symmetry studies. However, for antisymmetry, let us quote a definition to simplify our job further. Accordingly, "Operations of antisymmetry transform objects possessing two possible values of a given property from one value to the other".

Antisymmetry is the fundamental property of wave functions in the description of the electronic structure of atoms and molecules, but it is also much more common in our everyday world than we would first think. If we walk over to the campus of the University of Texas, at the northern foot of the Tower there are some Coke machines. There is a machine with the regular drink, another with the regular drink and yet another with the diet version. The operation transforming one

Fig. 7

Fig. 8

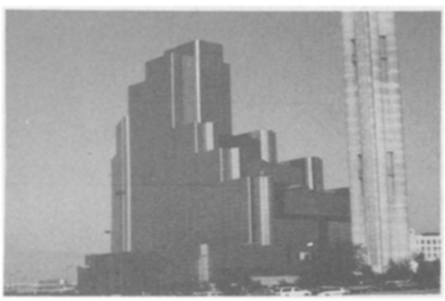

Fig. 9

Fig. 10

regular Coke machine into the other regular Coke machine is an identity operation (Fig. 11). However, the operation transforming one regular Coke machine into a diet Coke machine is an antiidentity operation (Fig. 12). In this case the property which has two possible values is the sugar content of the drink.

Antimirror reflections are common. Thus, for example, antisymmetry relates the two polyhedra colored in the opposite way (Fig. 13). Antisymmetry relates the girl and the elephant on the poster advertising a dancing class in Vienna (Fig. 14). An antisymmetry plane was thought to separate two world leaders in 1984 (Fig. 15), whereas a similar double portrait in 1988 might have used a simpler symmetry operation. Coloring changes express antisymmetry of a prize-winning Moscow poster from 1987 referring to Perestroika (Fig. 16).

Fig. 11

Fig. 12

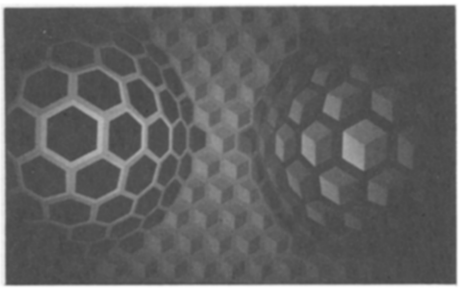

Fig. 13

Fig. 14

Human relations are full of symmetry and even fuller in antisymmetry. Hungarian author Karinthy has interpreted the eternal puzzle of the man/woman relationship by antisymmetry:

"Man and woman.
No wonder, they can't understand each other.
Each wants something different.
The man wants the woman,
The woman wants the man."

And now about Koestler. One of his major works is entitled *The Act of Creation*. Koestler made an attempt to inquire into the biological origins of mental creativity and he started by studying the phenomenon of laughter. He said,

"Humor is an elusive thing, so is the rainbow;
Yet the study of coloured spectra provided clues to the elementary structure of matter."

This elusiveness fits nicely the vagueness and fuzziness of our approach to symmetry in the nongeometrical world. Koestler has introduced the concept of so-called *bisociation*. Bisociation may be considered the antisymmetric partner of association. According to Koestler, the connection in association is made between thoughts on the same plane, whereas bisociation refers to such connection of thoughts from different planes.

A story, as an example, will clarify the matter perhaps better than any definition:

"A Marquis at the court of Louis XIV enters his wife's boudoir and finds her in the arms of a Bishop. The Marquis then walks calmly to the window and goes through the motions of blessing the people in the street.

'What are you doing?'—cries his anguished wife.

'Monseigneur is performing my functions',—replies the Marquis,—'so I am performing his.'".

Fig. 15

Fig. 16

The common association of thoughts would call for the Marquis's killing the Bishop, or his own wife, or both. The Marquis's reaction is quite unexpected, though it follows a certain logic of its own and qualifies his action for bisociation.

Let me quote at this point from a lecture a couple of days ago by Professor Nat Bauld who told the story of bringing Michael Dewar, the world famous theoretical organic chemist, to the University of Texas 25 years ago. He remembered that the UT football team was No. 1 nationally that year and the Chemistry Department was very proud of it. Bringing Michael Dewar to UT, Bauld said, was the first step in building up a Chemistry Department of which the UT football team could be proud. The first reference to the football team here is a simple thought association, while the last one appears in a bisociation.

Speaking about creativity, bisociation of different techniques and different fields has often led to strikingly new discoveries in the history of the sciences. Thus, there may indeed be something to bisociation.

Let us, however, return to another of Koestler's antisymmetric associations:

> "The Prince travelling through his domains, noticed a man in the cheering crowd who bore a striking resemblance to himself. He beckoned him over and asked:
> 'Was your mother ever employed in my palace?'
> 'No Sire',—the man replied.—'But my father was.'"

Here simple association would have meant a simple yes since it was common practice for feudal lords to have bastards, whereas feudal ladies were not supposed to have bastards.

And a very simple antisymmetric association, i.e. bisociation after Koestler,

> "What is a sadist?
> A sadist is a person who is kind to a masochist."

From the classical definition of symmetry, i.e. correspondence in size, shape and position of parts that are on opposite sides of a dividing line or center, we have come a long, long way. It is up to our tolerance how far we wish to go. Again, it is our ability to geometrize, to smooth over rough edges, to make complete a picture that has missing parts, or in other words, to discover the inherent, underlying symmetries even where there is none according to more rigorous criteria. This ability is an important tool in scientific research, facilitating the observation of trends, general phenomena, common characteristics, relationships, and creating classifications. However, it is then not less important to see the differences, to feel the rough edges, to be able to distinguish.

Let me quote at this point Michael Faraday about his childhood, after the theoretical chemist Coulson:

> "Do not suppose that I was a very deep thinker, and was marked a precocious person. I was a lively imaginative person, and could believe in the Arabian Nights as easily as in the Encyclopedia. But facts were important to me, and saved me".

To this Coulson adds,

> "It is when symmetry interprets facts that it serves its purpose: and then it delights us because it links our study of chemistry with another world of the human spirit—the world of order, pattern, beauty, satisfaction. But facts come first. Symmetry encompasses much—but not quite all!".